

# Recursive Functions

**CS 8: Introduction to Computer Science, Winter 2019**  
**Lecture #15**

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# Administrative

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- HW 7 due today!
- Left to-do:
  - HW 8 for Wednesday
  - Lab 7 for today (by midnite)
  - Project for Thursday

# FINAL IS COMING!



- Material: **Everything!**
- Homework, Labs, Lectures, Textbook
- **Wednesday, 3/20 in this classroom**
- **Starts at 8:00 AM \*\*SHARP\*\***
- *Bring your UCSB IDs and arrive 10-15 minutes early*
- Duration: **3 hours long** (but really designed for 1.5 – 2 hours)
- Closed book: no calculators, no phones, no computers
- Allowed: 1 sheet (**single**-sided) of written notes
  - Must be no bigger than 8.5" x 11"
  - **You have to turn it in with the exam**
- **You will write your answers on the exam sheet itself.**



**STUDY  
GUIDE NOW  
ONLINE!**

# Lecture Outline

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- Recursive Functions
- Exercises

# How *Do* Functions Work?

- Consider these 3 functions and tell me: what is **demo(-4)** ?

```
def demo(x):  
    return x + f(x)
```

```
def f(x):  
    return 11*g(x) + g(x/2)
```

```
def g(x):  
    return -1 * x
```

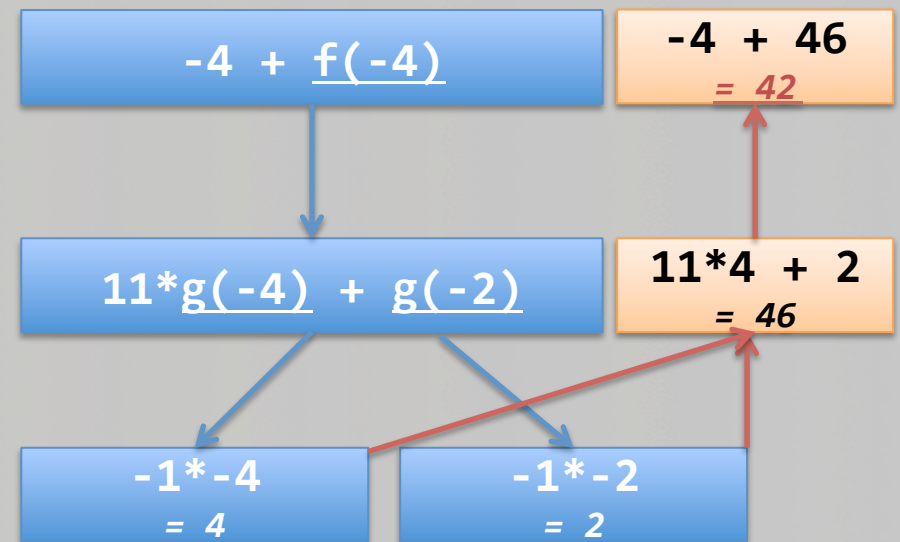
# How *Do* Functions Work?

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# What Keeps Track of All of This?!?

Ans: The Computer Memory Stack



(1) keeps separate variables for each function call...

(2) remembers where to send results back to...

*The stack is a special part of your computer's **memory**.*

*The **compiler** usually spells-out how the stack must be used with functions.*

**A child couldn't sleep,  
so her mother told a story about a little frog,  
who couldn't sleep,  
so the frog's mother told a story about a little bear,  
who couldn't sleep,  
so bear's mother told a story about a little weasel  
...who fell asleep.  
...and the little bear fell asleep;  
...and the little frog fell asleep;  
...and the child fell asleep.**



# Recursive Functions

- **Recursive: (adj.) Repeating unto itself**
- **A recursive function contains a call to itself**
- When breaking a task into subtasks, it may be that the subtask is a smaller example of the same task
- Just like functions-calling-functions, recursive functions make use of the stack

# Simple Example: Factorial Function

## Recall factorials:

$$2! = 1 * 2 ,$$

$$3! = 1 * 2 * 3 ,$$

$$4! = 1 * 2 * 3 * 4, \dots$$

$$N! = 1 * 2 * \dots * (N-1) * N$$

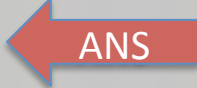
There's some repetition here... We could think of it as a loop  
(*how would you write that?*)

```
def factorial(n):  
    f = 1  
    for m in range(1, n+1):  
        f = f * m  
    return f
```

# Consider the Following...

```
def fac(N):  
    return N * fac(N-1)    # Yes, this is legal!  
print(fac(4))
```

**What happens when `fac(4)` is called?**

- A. It blows up! Does not compute! Does not compute!
- B. It returns the correct result (i.e. 24)
- C. The execution never stops (i.e. infinite loop) 
- D. It produces a return value but that value is incorrect (i.e. not 24)

## Just 'Cause It's Legal, Doesn't Mean It's Good Code!!!

```
def fac(N):  
    return N * fac(N-1)    # Yes, this is legal!
```

This goes on and on into an infinite loop!

Q: Why?

A: It's missing a “base case”  
(a.k.a a “stopping case”)

Q2: *What's a good “base case” here?*



# Base Case

```
def fac(N):  
    if N <= 1:  
        return 1  
    else:  
        return N * fac(N-1)
```

- Recursive functions should know **when to stop**
- There must be (at least) one ***base case***, and the recursive step must converge on a base case, otherwise you get an “***infinite recursion***”

# Under the Hood...

```
>>> fac(1)
```

I get:

1      # easy-peasy

```
>>> fac(5)
```

→ 5 \* fac(4)

→ 5 \* (4 \* fac(3))

→ 5 \* (4 \* (3 \* fac(2)))

→ 5 \* (4 \* (3 \* (2 \* fac(1))))

→ 5 \* (4 \* (3 \* (2 \* 1)))

= 120

*Every step, the new values are put into the **STACK** and kept track of by the computer*

```
def fac(N):  
    if N <= 1:  
        return 1  
    else:  
        return N * fac(N-1)
```

# Exercise

- What does **MyRecFun(3)** do?

```
def MyRecFun(n):  
    if n == 0:  
        return 2  
    else:  
        return 2*MyRecFun(n-1)
```



## Another Example: Mathematical Series

- Popular example: Fibonacci Series

$$F(n) = 1, 1, 2, 3, 5, 8, 13, \dots, F(n-1) + F(n-2)$$

- There's some repetition here...  
We could think of it as a loop also
- Or we could think of it as a recursive function!



# Fibonacci Recursion

- What is/are the BASE CASE(S)?
- What is the recursive formula?

```
def fibo(n):  
    if n == 0:  
        return 0  
    if n == 1:  
        return 1  
    else: # is this else necessary?  
        return fibo(n-1) + fibo(n-2)
```

DEMO  
TIME!

# YOUR TO-DOs

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- ☐ **HW8** (due on **Wednesday, 3/13**)
- ☐ **Lab7** (due on **Monday, 3/11**)
- ☐ **Project Assignment** (due on **Thursday, 3/14**)

**</LECTURE>**